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Semidefinite relaxation of optimal power flow

Abstract

The optimal power flow (OPF) problem seeks to optimize a certain objective, such as power loss, generation cost or user utility, subject to Kirchhoff's laws, power balance as well as capacity, stability and contingency constraints on the voltages and power flows. It is a fundamental problem that underlies many power system operations and planning. It is nonconvex and many algorithms have been proposed to solve it approximately. A new approach via semidefinite relaxation of OPF has been developed in the last few years.

In this tutorial, we present a bus injection model and a branch flow model, formulate OPF within each model, and prove their equivalence. The complexity of OPF formulated here lies in the quadratic nature of power flows, i.e., the nonconvex quadratic constraints on the feasible set of OPF. We characterize these feasible sets and design convex supersets that lead to three different convex relaxations based on semidefinite programming (SDP), chordal extension, and second-order cone programming (SOCP). When a convex relaxation is exact, an optimal solution of the original nonconvex OPF can be recovered from every optimal solution of the relaxation. We summarize three types of sufficient conditions that guarantee the exactness of these relaxations.

Finally, we extend the convex relaxations to multiphase unbalanced radial networks that are common in distribution
systems.

Biographical Sketch

Steven H. Low is a Professor of the Department of Computing \& Mathematical Sciences and the Department of Electrical Engineering at Caltech. Before that, he was with AT\&T Bell Laboratories, Murray Hill, NJ, and the University of Melbourne, Australia. He was a co-recipient of IEEE best paper awards, the R&D 100 Award, and an Okawa Foundation Research Grant. He is a Senior Editor of the IEEE Transactions on Control of Network Systems and the IEEE Transactions on Network Science & Engineering, is on the editorial boards of NOW Foundations and Trends in Networking, and in Electric Energy Systems, as well as Journal on Sustainable Energy, Grids and Networks. He is an IEEE Fellow and received his B.S. from Cornell and PhD from Berkeley, both in EE.